

Semester One Examination, 2017

Question/Answer booklet

**MATHEMATICS  
SPECIALIST  
UNIT 3**

**Section One:  
Calculator-free**

If required by your examination administrator, please place your student identification label in this box

Student Number: In figures

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In words

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Your name

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**Time allowed for this section**

Reading time before commencing work: five minutes

Working time: fifty minutes

**Materials required/recommended for this section**

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

**Important note to candidates**

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	12	12	100	98	65
<b>Total</b>					100

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Additional working space pages at the end of this Question/Answer booklet is for continuing an answer. If you use these pages, indicate at the original answer, the page number it is continued on and write the question number on the additional working space page.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you do not use pencil, except in diagrams.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

**Section One: Calculator-free****35% (52 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

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**Question 1****(6 marks)**

The position vector of  $R$ , the centre of a sphere with diameter  $\overline{PQ}$ , is  $2\mathbf{i} - \mathbf{k}$  and the position vector of  $Q$  is  $8\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ .

(a) Determine the position vector of  $P$ . (2 marks)

(b) Determine the vector equation of the sphere. (2 marks)

(c) The sphere intersects the  $y$ -axis where  $y = a$ . Determine the value(s) of the constant  $a$ . (2 marks)

**Question 2****(5 marks)**

A function is defined by  $g(z) = 2z^4 - z^3 + 7z^2 - 4z - 4$ .

(a) Show that  $z = 1$  and  $z = 2i$  are both zeros of  $g(z)$ . (2 marks)

(b) Determine all solutions to  $g(z) = 0$ . (3 marks)

**Question 3****(8 marks)**Simplify the following into the form  $x + iy$ .

(a)  $\frac{3}{2i} + 2i$ .

(2 marks)

(b)  $\frac{1}{(2-i)^2}$ .

(3 marks)

(c)  $(-\sqrt{2} + \sqrt{2}i)^6$ .

(3 marks)

**Question 4****(7 marks)**

The function  $f$  is defined by  $f(x) = \frac{1}{1-x}$ .

(a) Evaluate  $f(f(-1))$ . (1 mark)

(b) Determine and simplify an expression for  $f \circ f(x)$ . (2 marks)

(c) For  $f \circ f(x)$ , state the

(i) domain. (2 marks)

(ii) range. (2 marks)

**Question 5****(7 marks)**

(a) The equation  $2z^2 + 3z + 5 = 0$  has roots of  $\alpha$  and  $\beta$ . Determine the value of

(i)  $\alpha + \beta$ . (1 mark)

(ii)  $\alpha\beta$ . (1 mark)

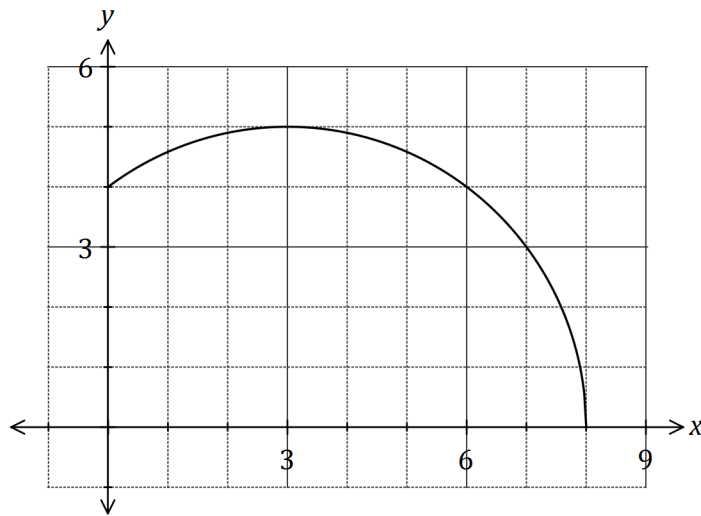
(iii)  $2\alpha^2 + 3\alpha + 5$ . (1 mark)

(b) Determine the values of the real constants  $a$  and  $b$  if  $z - 2 + i$  is a factor of  $z^3 + az + b$ . (4 marks)

## Question 6

(6 marks)

Let  $f(x) = \sqrt{16 + 6x - x^2}$ ,  $0 \leq x \leq 8$ . The graph of  $y = f(x)$  is shown below.



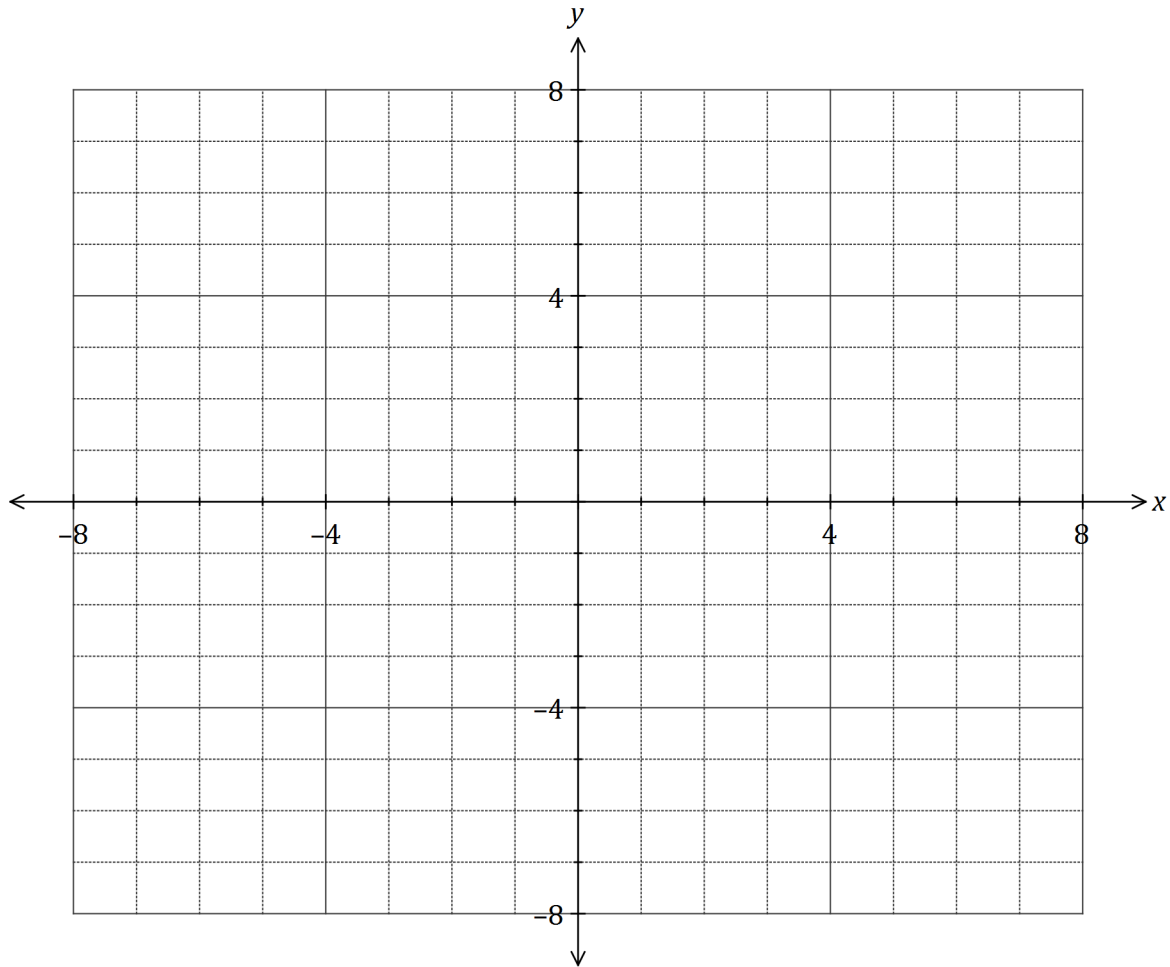
- (a) In order that  $y = f^{-1}(x)$  is a function, the domain of  $f$  must be restricted to  $k \leq x \leq 8$ . Explain why this restriction is necessary and state the minimum value of  $k$ . (2 marks)
- (b) Using the restriction from (a), determine the inverse function of  $f$  and its domain. (4 marks)



Question 7

(6 marks)

On the axes below, draw the graph of  $y = \frac{x^2}{x^2 - 2x - 3}$ , clearly showing key features and the behaviour of the curve near the asymptotes.



**Question 8****(7 marks)**

(a) Two of the solutions to the equation  $z^n = 1, n \in \mathbb{Z}^+$ , are  $z = \text{cis} \frac{\pi}{2}$  and  $z = \text{cis} \frac{\pi}{3}$ .

(i) State another solution to the equation.

**(1 mark)**

(ii) Determine, with reasons, the minimum value of  $n$ .

**(3 marks)**

(b) If  $z = \text{cis} \frac{\pi}{4}$ , determine the sum of the geometric series  $1 + z + z^2 + z^3 + \dots + z^{24}$ . Explain your answer. **(3 marks)**

Additional working space

Question number: \_\_\_\_\_

